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ACOUSTIC WAVE PROPAGATION IN A FLOWING LIQUID–VAPOUR MIXTURE

K. H. Ardron and R. B. Duffey†

Central Electricity Generating Board, Berkeley Nuclear Laboratories, Berkeley, Gloucestershire, England

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Abstract—The two-fluid equations describing transient nonequilibrium liquid–vapour flow have been used to derive a general dispersion relation for acoustic waves. The analysis is valid in principle for both dispersed and separated flow regimes. In contrast to previous work, the predicted sound speeds and attenuations depend only on measurable properties of the flow. The model can apply over a wide range of angular frequencies (up to 1–10^6 Hz for steam and water), under conditions where the scattering of waves by individual bubbles and droplets is unimportant. Predictions are made for sound speeds and attenuations in both bubbly and annular flow at low Mach numbers. Agreement of the theory with available data is shown to be reasonable.

1. INTRODUCTION

Determination of the speed and attenuation of acoustic waves in a vapour–liquid flow has applications in several important areas. These include the prediction of the onset of instability in parallel boiling channels (Ishii 1976), and the analysis of choked flows (Bouré *et al.* 1976). Acoustic wave measurements are of fundamental interest and have provided physical insight into the nature of multiphase systems in such diverse areas as plasma physics and crystallography.

In the published literature, two methods have been used for calculating sound wave properties in gas–liquid systems. One approach has been to consider the interaction of waves with individual bubbles and to use a statistical scattering theory to determine the propagation and attenuation of the wavefront (Morse & Feshbach 1953; Trammell 1962). This method predicts resonance absorption when the wavelength is comparable with the bubble size. At larger wavelengths, energy absorption by individual bubbles is not important. It is then possible to adopt a continuum theory in which the two-phase mixture is treated as a compressible fluid with suitably averaged properties (e.g. Mecredy & Hamilton 1972; Hsieh & Plesset 1961). The continuum model has the advantage that it can in principle readily provide a general dispersion relation valid for arbitrary flow regimes, including the effects of relative motion between the phases.

Mecredy *et al.* (1970) and Mecredy & Hamilton (1972) derived a detailed continuum model for sound wave propagation in vapour–liquid flow by using six separate conservation equations to describe the flow of the vapour and liquid phases. This is the so-called "two-fluid" representation of vapour–liquid flow, which allows for non-equilibrium mass, heat and momentum transfer between the phases. Results indicated that in a bubbly liquid high frequency waves travelled an order of magnitude faster than low frequency waves. However, the analysis of Mecredy & Hamilton (1972) contained the important assumption that evaporation and condensation rates were controlled by kinetic theory limitations. In consequence, predicted sound speeds were found to depend strongly on the assumed accomodation coefficients for molecular transfer. Since these coefficients are unknown functions of pressure, temperature and liquid cleanliness, etc. their results implied that sound speeds and attenuations could never be confidently predicted for a vapour–liquid system.

The object of this paper is to develop a model for sound-wave propagation in nonequili-

†Present address: EPRI, 3412 Hillview Avenue, P.O. Box 10412, Palo Alto, CA 94303, U.S.A.

brium vapour–liquid flows which predicts sound speeds and wave attenuations dependent only on measurable flow properties.

It is shown that in most practical cases of steam–water flow, mass-transfer rates are controlled by heat conduction in the liquid phase, and are not limited by kinetic theory as previously assumed. New acoustic wave dispersion relations are derived on this basis using the two-fluid conservation equations.

Frequency dependent sound velocities and attenuation rates are calculated for both dispersed and separated flow regimes with flow velocities up to a few metres/second. Results are compared with available data and with the predictions of scattering theory in the bubbly flow limit.

In addition to practical applications, this work provides a useful test of the hydrodynamic and constitutive equations inherent in the adopted flow model.

2. GOVERNING EQUATIONS

Two-fluid conservation equations

We use the two-fluid model of vapour–liquid flow, which incorporates six separate conservation equations for the flow of the gas and liquid phases, supplemented by equations relating interphase mass, energy and momentum transfer rates to averaged phase properties. This two-fluid model forms the basis for several recent analyses of transient fluid flow during water reactor depressurisation (e.g. Solbrig *et al.* 1976; Hancox *et al.* 1975; Harlow & Amsden 1975).

General equations of the three-dimensional two-fluid model are given by Ishii (1975). To simplify we consider a unidirectional flow parallel to the axis z of a constant area duct, and average the conservation equations across the duct area. Making the conventional simplifying assumption that the average of products of dependent variables over the duct area is identical to the product of averages the resultant one-dimensional equations for the conservation of mass momentum and energy of phase k are, respectively:

$$\frac{\partial}{\partial t}(\alpha_k\rho_k) + \frac{\partial}{\partial z}(\alpha_k\rho_k u_k) = \Gamma_k, \qquad [1]$$

$$\alpha_k\rho_k\frac{\partial u_k}{\partial t} + u_k\alpha_k\rho_k\frac{\partial u_k}{\partial z} + \alpha_k\frac{\partial p}{\partial z} = (u_{ki} - u_k)\Gamma_k + \tau_{ki} + \tau_{kw}, \qquad [2]$$

$$\alpha_k\rho_k T_k\frac{\partial S_k}{\partial t} + u_k\alpha_k\rho_k T_k\frac{\partial S_k}{\partial z} = \Phi_k + \Gamma_k(h_{ki} - h_k) + q_{ik}a_i + \tau_{ki}(u_{ki} - u_k) - \frac{\partial}{\partial z}\left(\alpha_k\epsilon_k\frac{\partial T_k}{\partial z}\right) + q_{wk}a_{wk} \qquad [3]$$

where u_k, p_k, T_k, S_k, h_k, ϵ_k and α_k denote respectively velocity, pressure, density, temperature, specific entropy, specific enthalpy, thermal conductivity, and volumetric concentration of phase k. [These are duct averaged values, which are also time averaged in the sense described by Ishii 1975)]. Γ_k is the rate of increase of mass of phase k per unit mixture volume due to phase change, and τ_{ki}, τ_{kw} represent the drag force on phase k per unit mixture volume due respectively to interfacial and wall shear. q_{ik} and q_{wk} are the heat fluxes into phase k across the interface and duct wall respectively. a_i is the interface area per unit mixture volume and a_{wk} is the duct wall area per unit mixture volume in contact with phase k. Φ_k denotes the viscous energy dissipation due to wall forces. The variables subscripted ki are properties of phase k in the neighborhood of the interface. Throughout the paper k will refer either to the gas phase $k \equiv G$, or the liquid phase $k \equiv L$.

Equations [1]–[3] treat the two-phase mixture essentially as a continuum with averaged properties, and hence can only describe the propagation of acoustic waves in the limit when details of the flow structure are unimportant. This means that for present purposes we must confine our attention to wavelengths which are large compared with bubble radii, average bubble separations, liquid film thicknesses etc. Sound waves of higher frequencies can excite resonances in individual bubbles, which cannot be described by a continuum theory (Minnaert 1936; Hsieh 1976). A comparison of the present theory with predictions of Trammell's (1962) scattering theory, which allows for resonant absorption by bubbles, is given later in the paper.

Constitutive and state equations

In order to integrate [1]–[3] additional relations are required to define conditions at the interface and to express the interphase mass energy and momentum transfer in terms of the bulk phase properties. These are provided by the following assumptions, applicable to vapour–liquid flow.

(i) The phase pressures are equal

$$p_L = p_G = p. \qquad [4]$$

Outside the region of bubble resonance differences in the phase pressures can only arise because of surface forces, which are usually negligible. For example, in the case of boiling water at atmospheric pressure [4] is accurate to within 1% if the bubble size exceeds 0.1 mm.

(ii) The liquid adjacent to the interface is saturated i.e.

$$T_{iL} = T_{\mathrm{SAT}}, \qquad [5]$$

where T_{SAT} is the saturation temperature at the system pressure. The justification for this important condition is given in appendix A.

(iii) Evaporation and condensation rates are controlled by heat transfer from the *liquid* to the interface. This assumption, which is usual in bubble growth analysis, is justified because q_{iG} is typically only a few per cent of q_{iL} when the interface is at the system saturation temperature. Neglecting q_{iG} and the small shear work term in an interfacial energy balance gives:

$$\Gamma_L = -\Gamma_G = q_{iL}a_i/h_{GL} \qquad [6]$$

where h_{GL} is the latent heat.

The magnitude of the heat flux in a periodic temperature field is given in section 3 below.

(iv) The mutual drag force per unit volume τ_{ki} is assumed to be a function of the relative velocity between the phases, u_r, and its time derivative, $\dot{u}_r$,

$$\tau_{ki} \equiv \tau_{ki}(u_r, \dot{u}_r); \qquad u_r = u_G - u_L, \qquad [7]$$

(Newton's third law implies that the drag forces are equal and opposite so that $\tau_{Gi} = -\tau_{Li}$). τ_{ki} will consist generally of both inertial and viscous components.

(v) The velocity field is continuous across the interphase boundary, with the interface velocity equal to the bulk velocity of the liquid phase,

$$u_{iG} = u_{iL} = u_L. \qquad [8]$$

This is reasonable for separated flows when $(u_G - u_L)$ can be large. In the bubbly flow regime the interface velocity will be closer to the bulk velocity of the gas u_G. However in this case the large mutual drag forces ensure that $u_L \simeq u_G$: thus the use of [7] introduces only a small error.

(vi) The duct wall is everywhere wetted so that:

$$a_{wG} = 0. \qquad [9]$$

(vii) It is assumed that the vapour obeys the perfect gas state equation

$$p = r\rho_G T_G, \qquad [10]$$

where r is the gas constant per unit mass. Liquid compressibility is retained in an approximate way by adopting a simplified liquid state equation which neglects the thermal expansivity of the liquid:

$$\mathrm{d}p/\mathrm{d}\rho_L = c_L^2, \qquad [11]$$

Here c_L^2 is the liquid sound speed, assumed independent of temperature.

3. LINEARISATION OF EQUATIONS, AND DERIVATION OF DISPERSION RELATION

Propagation of small periodic disturbances

We consider the motion of small amplitude monochromatic waves of frequency ω, wave-number k, travelling along the duct axis z. In the disturbed flow oscillations in the primary variables are given by:

$$\psi = \psi_0 + \psi' \exp i(\omega t - kz) \qquad [12]$$

where $\psi \equiv p$, T, ρ, u, α, etc. ψ_0 is the value of ψ in the undisturbed flow and $\psi' \ll \psi_0$. Terms of higher than first order in the primed variables are assumed negligible.

It is assumed that in the unperturbed (but not the perturbed) flow the phases are in equilibrium with each other, and with the duct walls.

Linearised transfer equations

The rates of interphase heat, mass, and momentum transfer will all be perturbed during a wave cycle and changes in each of these quantities will influence the velocity and attenuation of an acoustic disturbance. The transfer rates during the cycle are given as follows:

(a) *Heat transfer.* Since the undisturbed flow is in thermal equilibrium the unperturbed heat fluxes are zero so that $q_{ik0} = q_{wk0} = 0$. During the wave passage the fluxes across the interface and wall are:

$$q_{iL} = q'_{iL} \exp i(\omega t - kz) \qquad [13]$$

and

$$q_{wk} = q'_{wk} \exp i(\omega t - kz). \qquad [14]$$

A restriction on frequency allows us to neglect the contribution of convection to q'_{iL}. This condition is obtained by noting that the ratio of the diffusive component of q'_{iL} to the convective component is of order the ratio of the thicknesses of the convective and diffusive boundary layers, δ_C and δ_D respectively. Now for a laminar boundary layer with zero relative velocity at the interface:

$$\delta_C \sim (\nu_L L_s/u_r)^{1/2}$$

where u_r is the relative velocity ν_L the liquid kinematic viscosity, and L_s is a characteristic length. Since $\delta_D \sim (D_L/\omega)^{1/2}$, where D_L is the liquid thermal diffusivity,

$$\delta_C/\delta_D \simeq \frac{\nu_L L_s \omega}{u_r D_L}.$$

For convection in the liquid to be ignored $\delta_C/\delta_D \gg 1$, which implies the frequency restriction:

$$\omega \gg D_L u_r/(\nu_L L_s). \qquad [15]$$

For dispersed flow L_s is of order the particle size and is small but u_r is generally close to zero; for separated flows where u_r is large, L_s is of order of the duct size, which is also large. In most practical cases of steam/water flow [15] does not impose a severe frequency restriction, and we are justified in ignoring convection entirely.

The conductive heat flux in the liquid can be calculated approximately by solving the one-dimensional Fourier equation in the liquid close to the interface. The amplitude of the heat flux perturbation is related to the amplitude of the pressure perturbation p' by (see Appendix B)

$$q'_{iL} a_i \simeq \omega_q p' \qquad [16]$$

where ω_q is a complex frequency defined by

$$\omega_q = \omega^{1/2} \epsilon_L D_L^{-1/2} a_i (\mathrm{d}T/\mathrm{d}p)_{\mathrm{SAT}} \exp(i\pi/4).$$

$(\mathrm{d}T/\mathrm{d}p)_{\mathrm{SAT}}$ is the gradient of the saturation line.

Equation [16] is valid provided the diffusive thermal boundary layer thickness $\delta_D \sim (D_L/\omega)^{1/2}$ is small compared with the bulk dimension of the liquid phase, that is to say only a small fraction of the liquid phase undergoes a temperature change. This condition can be stated as:

$$a_i \delta_D \ll (1-\alpha) \quad \text{or} \quad \omega \gg D_L a_i^2 (1-\alpha)^{-2} \qquad [17]$$

where $\alpha \equiv \alpha_G = 1 - \alpha_L$. For water $D_L \simeq 10^{-7}\ \mathrm{m^2\,s^{-1}}$, and since for conditions of practical interest ($\omega > 10$ Hz) $\delta_D < 0.1$ mm, it follows that [17] is usually satisfied, even for finely dispersed flows (a_i large). Because the thermal boundary layer is thin, there is a steep temperature gradient near the interface. Hence the assumption that the interface temperature is equal to the bulk liquid temperature is not appropriate for an analysis of acoustic wave propagation in vapour–liquid mixtures. This assumption is implicit in the non-equilibrium kinetic theory analysis of Mecredy & Hamilton (1972).

The heat transfer to the vapour q'_{iG} will be ignored in formulating the linearised energy equation for the vapour. This term determines whether the small amplitude pressure changes in the vapour will be effectively isothermal or adiabatic. Ignoring it only introduces errors of order $\sqrt{\gamma} - 1$ into the calculated sonic speed, γ being the ratio of specific heats.

Since we consider only those flow regions in which the pipe walls are completely wetted there is no direct heat transfer between the walls and the vapour. Heat transferred to the liquid phase is proportional to the fraction of the wetted perimeter in contact with that liquid actually undergoing a temperature change during a wave cycle; this is of order the fractional volume of liquid contained in the thermal boundary layers enclosing the vapour–liquid interface, $a_i \delta_D (1-\alpha)^{-1}$, and is very small provided frequency condition [17] is satisfied. Thus for conditions of present interest we may take

$$q_{wG} = q_{wL} = 0. \qquad [18]$$

(b) *Mass transfer.* During the wave passage it follows from [6], [13] and [16] that

$$\Gamma_G = -\Gamma_L = -\frac{\omega_q p'}{h_{GL}} \exp i(\omega t - kz). \qquad [19]$$

(c) *Momentum transfer.* For small periodic changes in the liquid and vapour velocities the interphase drag may be expanded in the linear approximation to give (c.f. [7]),

$$\tau_{Gi} = -\tau_{Li} = \tau_{Gi0} + \tau'(u_G' - u_L') \exp i(\omega t - kz); \qquad [20]$$

where

$$\tau' = \left(\frac{\partial \tau_{Gi}}{\partial u_G}\right)_{u_L} = -\left(\frac{\partial \tau_{Gi}}{\partial u_L}\right)_{u_G} = -\left(\frac{\partial \tau_{Li}}{\partial u_G}\right)_{u_L} = \left(\frac{\partial \tau_{Li}}{\partial u_L}\right)_{u_G}.$$

The form of τ_{Gi}, τ_{Li} depends on the flow regime. Specific examples are inserted for illustrative calculations given in section 4.

Wall shear effects will be ignored in formulating the general dispersion relations: hence we put τ_W, $\Phi = 0$. The effect of wall forces on acoustic wave propagation, which is usually small, is discussed below.

Linearised conservation equations

Substituting the perturbed forms[12] into the conservation equations [1], [2], [3] and using the perturbed transfer rates q_{iL}, Γ_G and τ_{Gi} given by [13], [16], [19] and [20] we obtain the following six linear homogeneous equations in six unknown primed quantities:

$$\left.\begin{aligned}
-ik\alpha\rho_G u_G' + i\omega_G\alpha\rho_G' + \omega_q h_{GL}^{-1} p' + i\omega_G\rho_G\alpha' &= 0\\
-ik(1-\alpha)\rho_L u_L' + [i\omega_L(1-\alpha)c_L^{-2} - \omega_q h_{GL}^{-1}]p' - i\omega_L\rho_L\alpha' &= 0\\
[i\alpha\rho_G\omega_G - \tau']u_G' + \tau' u_L' + [\omega_q h_{GL}^{-1}(u_L - u_G) - ik\alpha]p' &= 0\\
\tau' u_G' + [i(1-\alpha)\rho_L\omega_L - \tau']u_L' - i(1-\alpha)kp' &= 0\\
\xi u_G' - \xi u_L' + [i\omega_G\rho_G\alpha T S_G^p + \alpha\epsilon_G k^2 c_T^2/r\rho_G]\rho_G' + [i\omega_G\rho_G\alpha T S_G^\rho - \alpha\epsilon_G k^2/r\rho_G]p' &= 0\\
[i(1-\alpha)\rho_L C_{pL}\omega_L - (1-\alpha)\epsilon_L k^2]T_L' - \omega_q p' &= 0
\end{aligned}\right\} \qquad [21]$$

where C_{pL} is the liquid specific heat at constant pressure. These equations are, respectively, the continuity equations for the gas and liquid, momentum conservation equations for the vapour and liquid and the energy conservation equations for the vapour and liquid. Note that the vapour energy equations and the liquid continuity equation incorporate the linearised state equation for the vapour and liquid phases (see [10], [11]):

$$p' - c_T^2\rho_G' - r\rho_G T_G' = 0,$$

$$p' - c_L^2\rho_L' = 0.$$

For convenience we have dropped the zero subscripts from the unperturbed variables and used the additional notation:

$$\alpha = \alpha_G = (1 - \alpha_L),$$

$$S_G^p = \left(\frac{\partial S_G}{\partial \rho}\right)_p \quad \text{and} \quad S_G^\rho = \left(\frac{\partial S_G}{\partial p}\right)_\rho,$$

$$\xi = \tau_{Gi} + (u_G - u_L)\tau',$$

$$c_T^2 = \frac{p}{\rho_G},$$

$$\omega_G = \omega - u_G k; \qquad \omega_L = \omega - u_L k.$$

In deriving [21] we have introduced an additional simplifying approximation by ignoring terms in the density, pressure and entropy gradients of the undisturbed flow $(\partial \rho_{G0}/\partial z)$, $(\partial p_0/\partial z)$, (DS_{G0}/Dt_G) etc. An order-of-magnitude comparison shows that these terms only become important when the change in the velocity of the undisturbed flow over distances of order one wavelength is of order the sound velocity; we can therefore ignore them in most cases of interest.

General dispersion relation

Eliminating α' between the first two of [21] reduces the set to five linear equations. The secular equation of this set provides the dispersion relation for periodic disturbances. With a little algebraic manipulation this can be written in the determinantal form:

$$i(1-\alpha)\rho_L C_{pL}\omega_L - (1-\alpha)\epsilon_L k^2] \times \begin{vmatrix} -i\tau' & (1-\alpha)\rho_L\omega_L & -(1-\alpha)k & 0 \\ \alpha\rho_G\omega_G & \alpha\rho_G\omega_G + (1-\alpha)\rho_L\omega_L & \omega_q(u_L-u_G)/ih_{GL} - k & 0 \\ -k\omega_L\rho_L\rho_G\alpha & -k\rho_L\rho_G[\omega_G\alpha + \omega_L(1-\alpha)] & [\omega_G\omega_L\rho_G(1-\alpha)c_L^{-2} - (\omega_G\rho_G - \omega_L\rho_L)\omega_q/ih_{GL}] & \omega_G\omega_L\rho_L\alpha \\ \xi & 0 & [i\omega_G\alpha\rho_G TS_G^P - \alpha\epsilon_G k^2/r\rho_G] & [i\omega_G\alpha\rho_G TS_G^\rho + \alpha\epsilon_G k^2 c_T^2/r\rho_G] \end{vmatrix} = 0 \quad [22]$$

Note that the thermodynamic derivatives satisfy the following relations:

$$-S_G^\rho/S_G^p = c_s^2 = \gamma p/\rho_G \qquad TS_G^p = C_{vG}/(\rho_G r). \quad [23]$$

where C_{vG} and γ are respectively the specific heat at constant volume, and the specific heat ratio of the vapour phase.

Equation [22] is the general dispersion relation for periodic disturbances but is rather cumbersome in its present form. To simplify we ignore terms in the linearised energy equations due to axial heat diffusion which for steam–water flows are negligible for frequencies much smaller than 1 GHz, [they do of course dominate the limit $\omega \to \infty$ (see section 4 below)].

Equation [22] then has six independent solutions $k_i(\omega)$ corresponding to two "path waves" advancing with the bulk phase velocities u_G and u_L, and two "composite waves" which are a mixture of acoustic and kinematic perturbations. A useful analytic approximation to the acoustic wave dispersion relation can be made by considering flows at low Mach numbers u_G, $u_L \ll \omega/k$ such that ω_G, $\omega_L \simeq \omega$. The composite waves then degenerate into two path waves and two acoustic waves. Multiplying out the determinant in [22], using [23], the wavenumbers of the acoustic disturbances can be shown to be roots of the quadratic equation:

$$\{i\omega^2\rho_L\rho_G\alpha(1-\alpha)[\rho_L\alpha + \rho_G(1-\alpha)] - \omega\rho_L\rho_G\tau'\}k^2 + \left\{\frac{\gamma-1}{c_s^2}\xi\omega^2(1-\alpha)\rho_L\rho_{GL} + k_q\rho_L\rho_G\omega[(1-\alpha)\alpha\omega\rho_L + i\tau']\right\}k + i\frac{\gamma-1}{c_s^2}\xi\omega^2(1-\alpha)\rho_L^2 k_q - i\omega^3\left[\frac{\rho_G(1-\alpha)}{c_L^2} + \frac{\rho_L\alpha}{c_s^2} - \frac{\omega_q\rho_{GL}}{ih_{GL}\omega}\right][i\tau'\rho_m + (1-\alpha)\alpha\rho_L\rho_G\omega] = 0, \quad [24]$$

where $\rho_m = \rho_G\alpha + \rho_L(1-\alpha)$ is the two-phase mixture density and $k_q = \omega_q(u_G - u_L)h_{GL}^{-1}$ is a characteristic wave number associated with the momentum transfer during the phase change. ρ_{GL} denotes the density difference $\rho_G - \rho_L$.

Effects of pipe wall friction

The effects of wall shear on wave propagation can be examined by introducing terms of the form $\tau_w = \frac{1}{2}f_w\rho u^2/D_h$ and $\Phi = u\tau_w$ into the momentum and energy equations [2] and [3], where D_h is the hydraulic diameter and f_w is the D'Arcy wall friction factor. Linearising the resultant equations for a homogeneous flow ($u_G = u_L$) shows that wall shear increases the calculated wave speed by an amount of order $f_w^2u^2/8\omega^2D_h^2$. For flows of a few meters per second, for which [24] is applicable, this represents only a small percentage correction in c, and we are justified in ignoring wall shear effects in the above derivation.

4. RESULTS AND DISCUSSION

Equation [24] has two complex solutions in k corresponding to waves moving upstream and downstream. When there is no interphase relative motion in the unperturbed flow the solutions are equal but of opposite sign. However, when u_r is finite the two solutions differ slightly in magnitude. Denoting the solutions by k_1, k_2 the velocities and attenuations of these waves are given by, respectively,

$$c_{1,2} = \omega/\{\text{real part of } k_{1,2}\}, \tag{25}$$

$$\eta_{1,2} = \{\text{imaginary part of } k_{1,2}\}. \tag{26}$$

Numerical examples

Results of illustrative calculations for c and η are plotted in figure 1 for a finely dispersed bubbly flow (bubble radius $R_b = 0.25$ mm) for various low void fractions at three pressures up

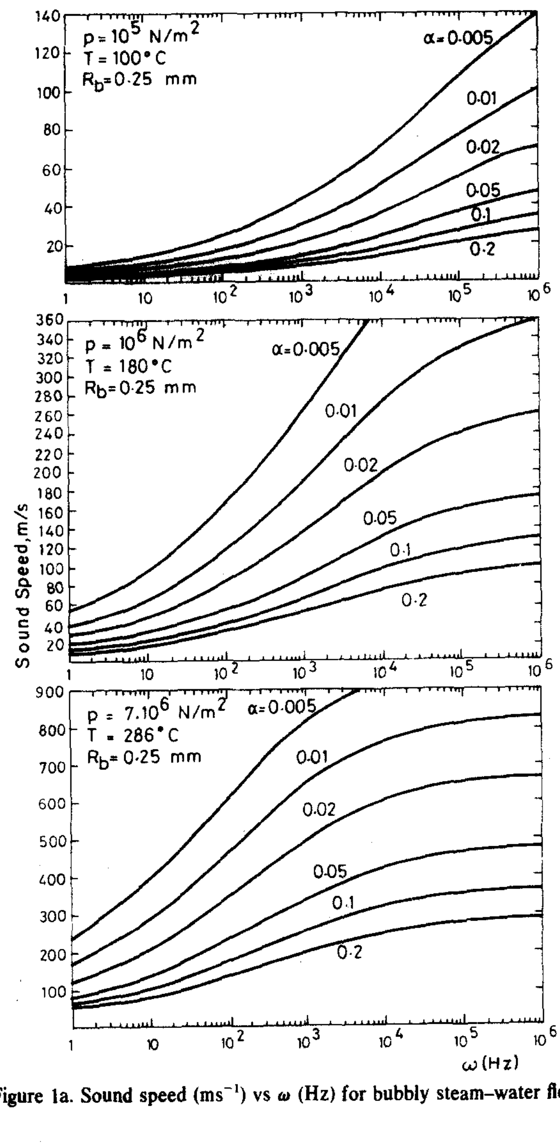

Figure 1a. Sound speed (ms^{-1}) vs ω (Hz) for bubbly steam–water flow.

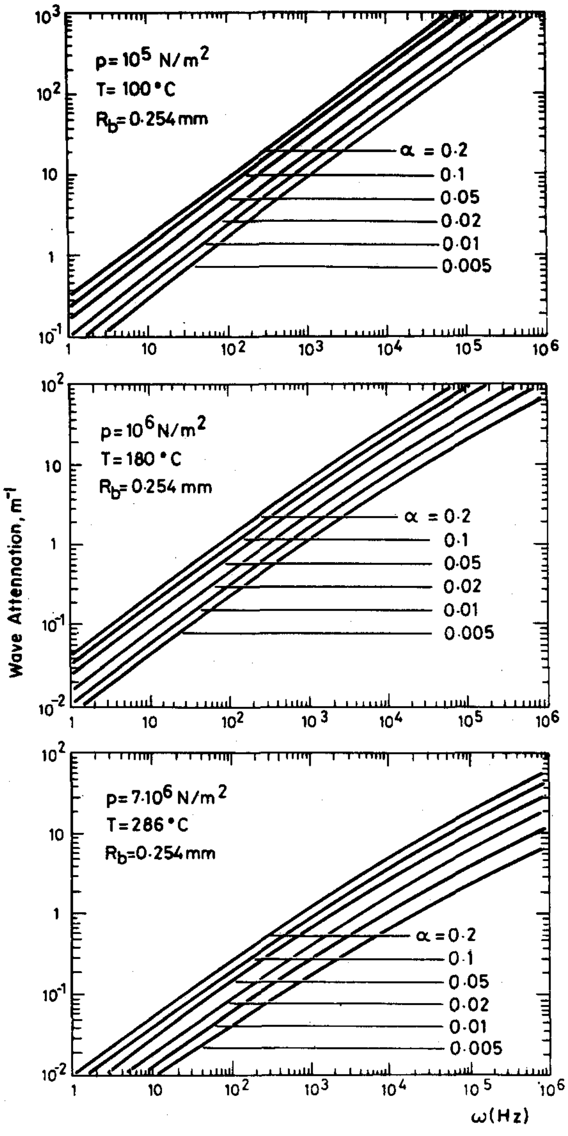

Figure 1b. Wave attenuation (m^{-1}) vs ω (Hz) for bubbly steam–water flow.

to 70 bar. The unperturbed phase velocities have been assumed to be equal ($u_{G0} = u_{L0}$). The interfacial area and drag relationships used in these calculations are standard forms given in table 1.

It can be seen from figure 1(a) that because of non-equilibrium effects the sound speed is strongly dependent on the frequency of the wave, tending to a finite limit at high frequencies (see [31] below). Only at low frequencies is the sound speed near the homogeneous equilibrium value. Figure 1(b) shows that the attenuation is large above $\sim 10^3$ Hz varying approximately as (angular frequency)^n, where n lies between 0.6 and 0.75 and depends on the pressure.

Figure 2 shows results of similar calculations for a high void fraction annular flow. The adopted interfacial area and drag relationships are again listed in table 1. Results are given for vapour/liquid relative velocities of 1 and 2 m/s in a tube of 20 mm diameter. In all cases the sound speed rapidly increases with frequency to the so-called "stratified" limit, when momentum transfer effects become negligible (c.f. [29] below). The attenuation is typically an order of magnitude less than for bubbly flow.

Table 1. Expressions for interfacial area concentration, a_i, and interphase drag, τ_{Gi}, used for bubbly and annular two-phase flow

Bubbly flow	Derivation	Annular flow	Derivation
$a_i = 3\alpha/\bar{R}_b$	Derived assuming a uniform distribution of spherical bubbles of mean radius $\bar{R}_b$.	$a_i = \frac{2}{D_h}(1+\alpha)$	Derived assuming a liquid film of uniform thickness.
$\tau_{Gi} = -9\alpha\mu_L u_r/2\bar{R}_b^2 - \frac{\alpha}{2}\rho_L \frac{1+2\alpha}{1-\alpha}\left[\frac{\partial u_r}{\partial t} + u_G\frac{\partial u_r}{\partial z}\right]$	Obtained from the expression for the drag on a single sphere given by Brodkey (1968). The first term is the Stokes viscous drag, and the second an inertial drag due to the virtual mass of the bubble.	$\tau_{Gi} = -\frac{f_i}{2}\rho_G a_i u_r^2$ where $f_i = 0.005(1+\epsilon)$ $\epsilon \sim 75(1-\alpha)$	Derivation discussed by Wallis (1969).
$\tau'(=\partial\tau_{Gi}/\partial u_G) = -9\alpha\mu_L/2\bar{R}_b^2 - i\frac{\alpha}{2}\frac{1+2\alpha}{1-\alpha}\rho_L\omega_G$	Obtained using the perturbations (10), μ_L being liquid viscosity.	$\tau'(=\partial\tau_{Gi}/\partial u_G) = -f_i\rho_G a_i u_r$	

High frequency limits of solution

It is interesting to consider further the high frequency limits of the solutions for separated and dispersed flows.

(a) *Separated flow*. For this case the drag τ_{Gi} contains no acceleration dependent terms. Consequently τ' is independent of ω so that as ω, $k \to \infty$ all terms contributed by momentum, mass and heat transfer appearing in [22] become negligible. Also the axial heat diffusion terms in the fourth row of the determinant become large compared with the terms in the entropy derivatives. Evaluating the limit we obtain a dispersion relation for acoustic waves of the form:

$$\alpha\rho_L\omega_L^2 + (1-\alpha)\rho_G\omega_G^2 - \omega_L^2\omega_G^2\left\{\frac{(1-\alpha)\rho_G}{c_L^2} + \frac{\alpha\rho_L}{c_T^2}\right\}k^{-2} = 0 \qquad [27]$$

where $c_T = \sqrt{p/\rho_G}$ is the isothermal sound velocity in the gas. When there is no interphase relative motion in the undisturbed flow, so that $\omega_G = \omega_L = \omega - uk$, [27] has real solution for k given by:

$$\omega/k = u, \qquad \omega/k = u \pm c_{ST\infty}, \qquad [28]$$

where

$$c_{ST\infty} = \{\rho_L\alpha + \rho_G(1-\alpha)\}^{1/2}\left\{\frac{(1-\alpha)\rho_G}{c_L^2} + \frac{\alpha\rho_L}{c_T^2}\right\}^{-1/2} \qquad [29]$$

is the well-known so-called stratified sound speed for the mixture, obtained for example by Grolmes & Fauske (1971) by neglecting interphase mass, heat and momentum transfer.

Lyczkowski *et al.* (1975) show that the wave velocities given by [28], $\omega/k = u$, $u \pm c_{ST\infty}$ are characteristic velocities for the system of linear differential equations [1], [2], [3] when the velocities of the phases are equal, and the transfer terms contain no time or spatial derivatives. In the flow of a perfect gas these characteristic velocities can be identified with the propagation velocities of small disturbances. In a separated two-phase flow in which the phase velocities are equal, the characteristic velocity evidently corresponds to the high frequency limit of the acoustic wave velocity. Inspection of figures 1 and 2 show that sound speed increases with the frequency, which suggests that within the framework of the continuum model the characteristic velocities are the maximum velocities at which information can be transmitted upstream or downstream.

It is interesting to note that the condition that the characteristic velocity is zero has been

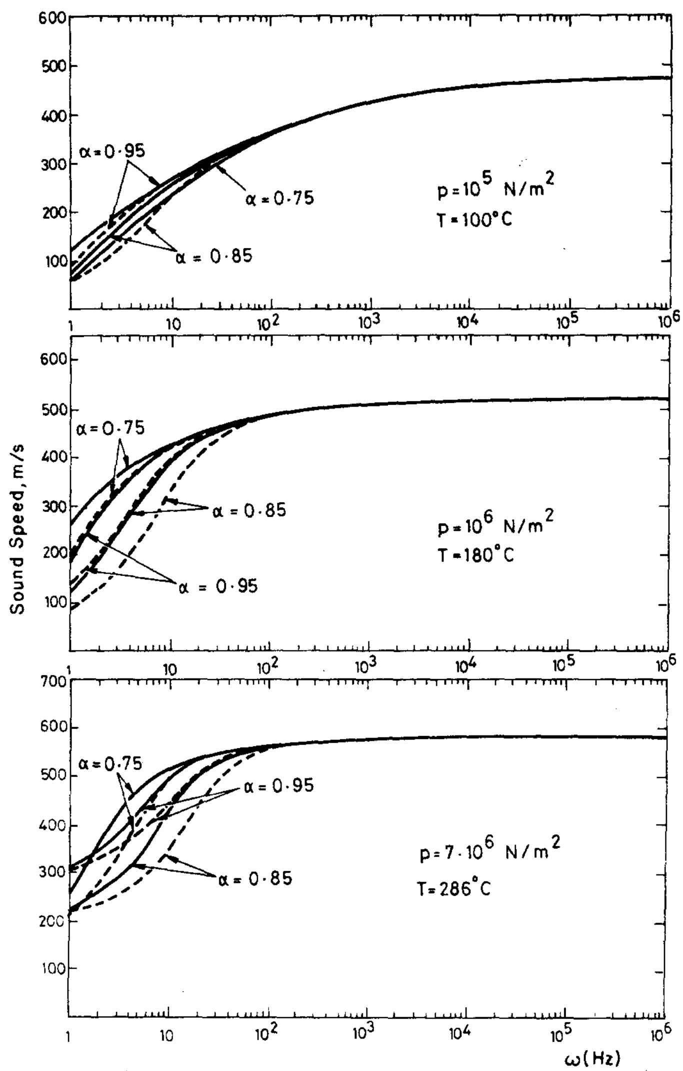

Figure 2a. Sound speed (ms^{-1}) vs ω (Hz) for annular steam water flow. The solid line is for $u_G - u_L = 1\ ms^{-1}$ and the broken line for $u_G - u_L = 2\ ms^{-1}$.

traditionally adopted as a choking criterion in one-dimensional models of both single and two-phase flows. The above considerations tend to confirm the intuitive argument that in the continuum model for two-phase flow this condition describes the state where the flow velocity is sufficient entirely to prevent the transmission of information upstream. This result, which is known to be valid for ideal gas flow, also holds for dispersed flows (see (b) below).

(b) *Dispersed flow.* For this case the drag τ_{Gi} depends on the acceleration of the gas phase through virtual mass terms. Consequently τ' depends linearly on ω_G, and momentum exchange between the phases is important even in the high frequency limit. Since in a dispersed flow $u_{G0} \simeq u_{L0}$ we can write $\omega_G \simeq \omega_L \simeq \omega - uk$ for this case. Using this fact, and the expression for τ' given for bubbly flow in table 1, [22] becomes, in the limit $\omega \to \infty$:

$$(\omega - uk)^2\{(\omega - uk)^2 - k^2c_{D\infty}^2\} = 0 \qquad [30]$$

where using the fact that $\rho_G/\rho_L \ll 1$, $c_{D\infty}^2$ can be expressed to a good approximation as:

$$c_{D\infty}^2 = \frac{\rho_G\rho_L}{\rho_m}\left\{\frac{2\alpha(1-\alpha)^2}{1+2\alpha} + 1\right\}\left\{\frac{\rho_G(1-\alpha)}{c_L^2} + \frac{\rho_L\alpha}{c_T^2}\right\}^{-1} \qquad [31]$$

which is similar to the form derived by Mecredy & Hamilton (1972).

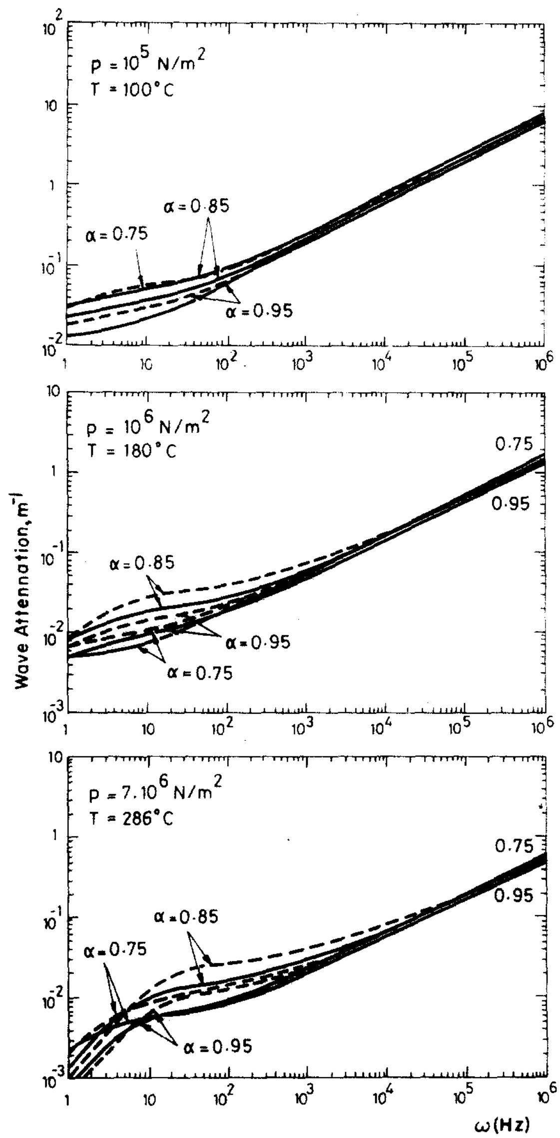

Figure 2b. Wave attenuation (m^{-1}) vs ω (Hz) for annular steam-water flow. The solid line is for $\mu_G - u_L = 1\ ms^{-1}$ and the broken line for $u_G - u_L = 2\ ms^{-1}$.

Equation [30] has real solutions in k given by:

$$\omega/k = u, \qquad \omega/k = u \pm c_{D\infty}. \tag{32}$$

We note that since τ_{Gi} contains terms in the spatial and time derivatives the characteristic velocities implied by [21] are different in this case than for separated flow; in fact they can again be shown to correspond to the limiting sound speeds in the fluid obtained from the second of the solutions [32]. Thus the discussion given in (a) above applies also to this dispersed flow case.

Comparison of present theory with predictions of other models

Figure 3(a) compares sound velocities calculated from [24] with the theoretical results of Mecredy & Hamilton (1972) who also used a continuum two-fluid model but, it will be recalled, neglected the temperature gradients at the interface and calculated mass-transfer rates using kinetic theory. The large differences between the predictions of the two models are a direct consequence of these inappropriate interphase mass and energy transfer assumptions. Figure

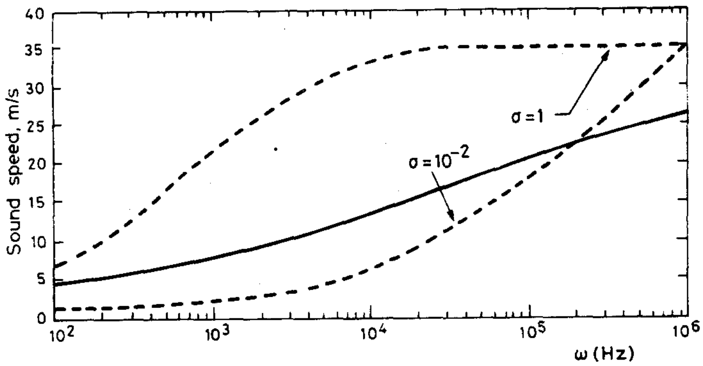

Figure 3a. Comparison between the present theory (solid line) and theory of Mecredy & Hamilton (1972) (broken line). Calculations are for a bubbly steam water mixture, $p = 10^5\ \mathrm{N/m^2}$, $\alpha = 0.2$, $R_b = 0.25$ mm.

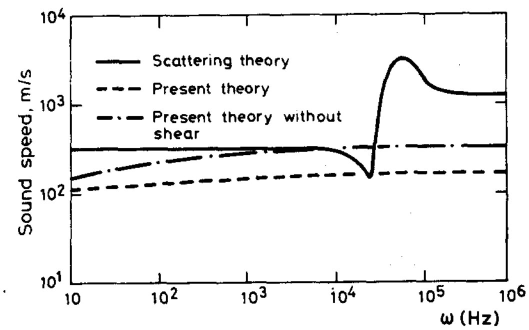

Figure 3b. Comparison between present theory and predictions of the scattering theory of Trammell (1962) as calculated by Kielland (1967).

3(a) highlights the strong dependency of Mecredy & Hamilton's results on the highly uncertain accommodation coefficient, σ.

A comparison of the present model with the predictions of Trammell's (1962) scattering theory (calculated by Kielland (1967)) for a bubbly mixture, is shown in figure 3(b). The most noticeable differences here are the presence of the volume resonance near 3.10^4 Hz and the prediction of the scattering model that the sound speed tends to the liquid sound speed, c_L, in the high frequency limit. These trends have been qualitatively confirmed by experiment (see below). Both analyses would be expected to give similar results in the low frequency limit, when the wavelength is such that $\lambda \gg \bar{R}_b$. However, Trammell's theory does not allow for interphase momentum transfer. The effect of artificially neglecting shear in the present analysis ($\tau' \rightarrow 0$) can be seen in figure 3(b) to significantly increase the agreement between the theories below the resonance frequency.

Morioka & Matsui (1975) developed a model of acoustic wave propagation in a stratified flow which, while neglecting interphase mass, heat and momentum transfer, retains velocity components perpendicular to the duct axis. Their calculations showed that the phase velocity of the dominant mode was very close to the stratified speed predicted by a one-dimensional model ([29]) when the interfacial transfer terms are set to zero. This tends to support the use of a one-dimensional approximation for calculating wave-speeds in separated flows.

Comparison with experiments

Weisman *et al.* (1976) have measured the phase velocity of small amplitude sinusoidal pressure fluctuations in a flowing vapour–liquid mixture of a fluorocarbon (Freon-113). Because the wave frequency was low ($\omega = 7.85$ Hz) the continuum condition ($\lambda \gg R_b$) applies, and since the thermal diffusivity is small, the thin thermal boundary layer condition [17] is also satisfied. Thus the present model is applicable for the test conditions. Bubble sizes were not measured in the experiments: however, use of an empirical relation given by Wallis (1969) (of the form $R_b \sim (s/g\rho_L)^{1/2}$ where s is surface tension and g gravitational acceleration) suggests that under the stated conditions bubble radii were ~1 mm. The present theory is compared with the 95% confidence limits of the data in figure 4: agreement is generally within the experimental scatter, providing encouraging support for the model.

Feldman *et al.* (1971) and Kokernak & Feldman (1972) have reported some measurements of sound velocity in bubbly stream–water mixtures and in a Freon-12 liquid–vapour mixture. Because these experiments were directed towards investigating bubble resonance effects, the present model would be expected to apply only over a small part of the measured frequency range.

The present theory is compared with the Freon data in figure 5(a). Agreement with the data for small bubble sizes is reasonable, within the region of validity of the model. At high frequencies, the sound speeds tend to the sound speed in the liquid ($c_L \simeq 500$ m/s) as predicted by scattering theory. The data for large bubble sizes appeared to show a low frequency resonance which was inconsistent with the stated bubble dimensions, as pointed out by Kokernak & Feldman (1972). This suggests the presence of bubbles larger than the stated size and may account for the relatively poor agreement for this case. A similar difficulty was found by Kokernak & Feldman in fitting these results to a scattering model.

The present theory is compared with the steam–water data in figure 5(b). Comparisons here are somewhat inconclusive since there were substantial uncertainties in the reported void fraction which was believed to lie in the range $\alpha = 0.03$–0.3%. Best agreement with the data is obtained using the present model if a void fraction of 0.03% is adopted. Feldman *et al.* (1971) found that scattering theory also gave satisfactory results if this value were assumed for α. Thus, these data are not inconsistent with the present analysis.

Several investigators have attempted to resolve the mechanism of sound-wave transmission in liquid–vapour mixtures by measuring the velocity of the leading edge of large amplitude pressure pulses (Semenov & Kosterin 1964; Karplus 1961; Grolmes & Fauske 1969). These data are very difficult to interpret since in the reported tests the pulses have been observed to broaden, with different points within the pulse width travelling with different speeds. This broadening is presumably attributable to combined effects of dispersion and departures from linear behaviour. It seems clear that without a detailed analysis of these effects (as given by Morioka & Matsui (1975) for an idealised stratified flow) velocity measurements of this type cannot be used to provide useful data about the transfer processes responsible for the dispersive properties of two-phase flows, which are of interest here. Experiments with monochromatic sound waves avoid these problems of interpretation.

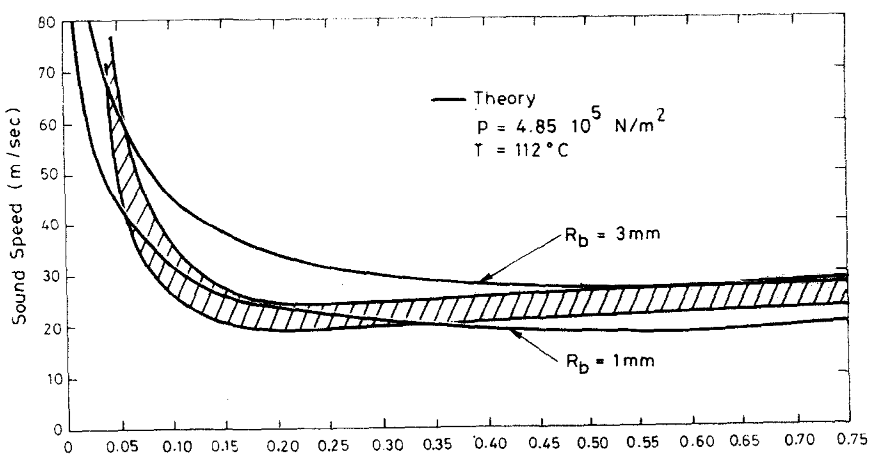

Figure 4. Velocity of sound in Freon-113 at $\omega = 7.85$ Hz. The shaded region is the range of the data of Weisman *et al.* (1976).

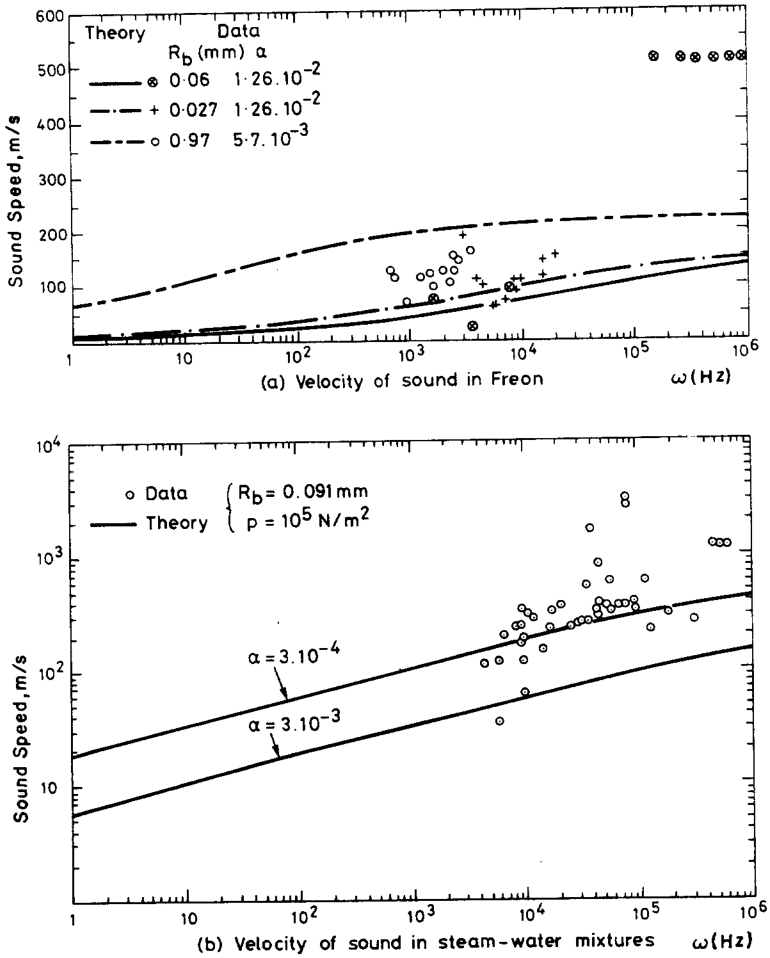

Figure 5. Velocity of sound in liquid–vapour mixtures. Data are from Feldman *et al.* (1971) and Kokernak & Feldman (1972).

5. DISPERSION RELATIONS PREDICTED BY SIMPLIFIED ONE-DIMENSIONAL MODELS OF TRANSIENT VAPOUR-LIQUID FLOW

Equation [24] is derived from a detailed flow model allowing for non-equilibrium mass, heat and momentum transfer. A comparison with dispersion equations obtained from simpler models proposed for transient two-phase flow (e.g. for water-reactor loss-of-coolant analysis) gives insight into the consequences of the simplifying assumptions incorporated into these models.

Figure 6 compares wave-speeds given by the full solution with predictions of a homogeneous model in which interphase relative motion is forbidden (obtained by putting $\tau' = \infty$ in [24]). It is seen that the effect of this particular restrictive assumption is negligible in low-quality bubbly flows at all frequencies because of the large effective bubble inertia.

Also shown in the figure are predictions of the homogeneous thermal equilibrium flow model (which assumes infinite rates of interphase mass heat and momentum transfer) and a homogeneous frozen-composition model (assuming equal phase velocities and zero heat and mass transfer). These latter models predict wave propagation without dispersion or attenuation and only agree with the present theory in the limits of low and high frequencies.

An interesting attempt to correct the homogeneous equilibrium model for finite heat and mass-transfer rates has been described by Kroeger (1976) and Bauer *et al.* (1976) who propose an arbitrary relaxation equation for quality of the form:

$$\frac{\partial X}{\partial t} + u\frac{\partial X}{\partial z} = \frac{X_{EQ} - X}{\Theta}$$

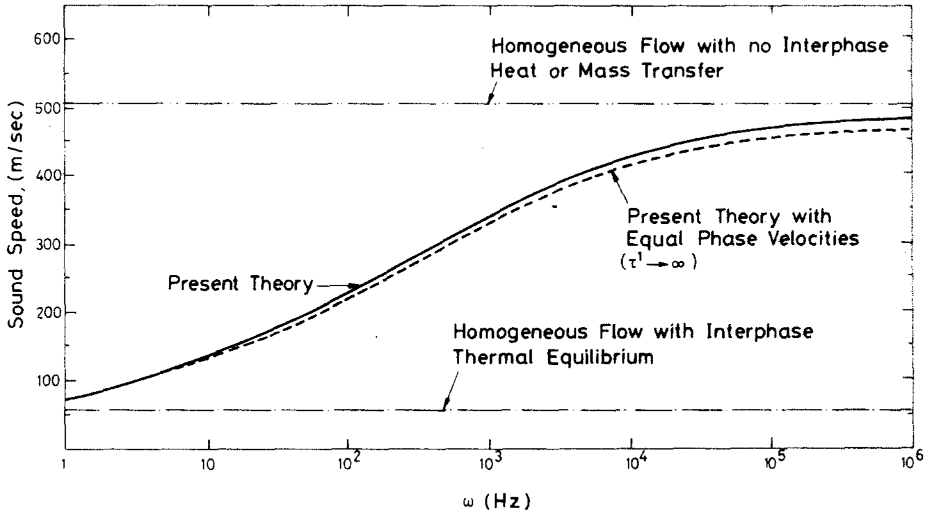

Figure 6. Comparisons of sound speeds predicted using various simplifying assumptions. Calculations are for bubbly steam–water flow $p = 7.10^6$ N/m^2, $\alpha = 0.05$, $R_b = 0.25$ mm.

where X_{EQ} is the quality that would exist were the phases always in thermal equilibrium. With the continuity and momentum equations for homogeneous two-phase flow, this equation leads to the dispersion relation:

$$\omega^2/k^2 = (1 + i\omega\Theta)(1/c_{EQ}^2 + i\omega\Theta/c_{NEQ}^2)^{-1} \qquad [33]$$

where c_{EQ} is the usual homogeneous thermal equilibrium sound speed and $c_{NEQ}(=\sqrt{\gamma p/\alpha\rho_m})$ is two-phase sound speed for homogeneous flow in the absence of heat or mass transfer. Wave speeds predicted by [33] are compared with the full solution in figure 7. It is seen that although [33] predicts the correct behaviour in the limits $\omega \to 0$ and $\omega \to \infty$, good agreement with the full theory over the range of intermediate frequencies cannot be obtained by adjusting the characteristic relaxation time, Θ.

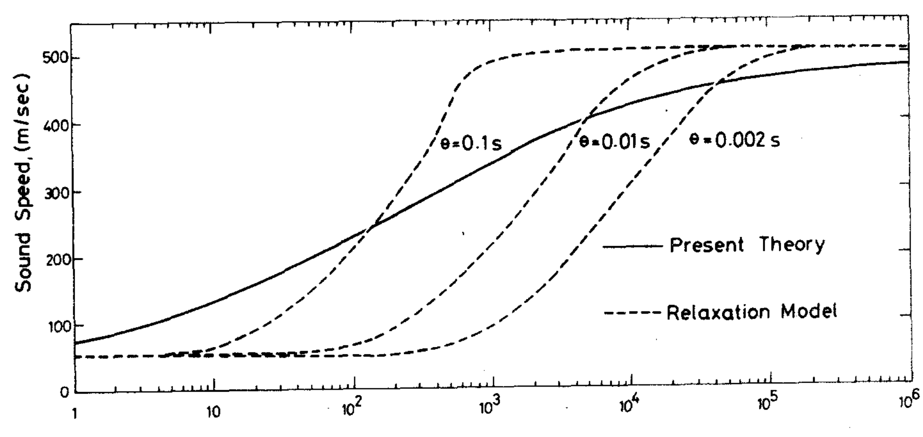

Figure 7. Comparison of present theory with the relaxation model proposed by Kroeger (1976). Calculations are for bubbly stream–water flow, $p = 7.10^6$ N/m^2, $\alpha = 0.05$, $R_b = 0.25$ mm.

6. CONCLUSIONS

One-dimensional two-fluid equations have been used to derive a new dispersion relation for non-equilibrium vapour–liquid flows. The model, which is valid if the sound wavelength is large compared with the scale of the flow structure, is applicable over a wide frequency range.

Predicted sound speeds and wave attenuation rates depend only on experimentally measurable properties of the flow.

By taking conventional formulations for interfacial area and shear, sound speeds and attenuations are calculated both for dispersed and separated flows at low Mach numbers. For low quality bubbly flows results of the theory are consistent with predictions of a scattering model, and are in agreement with available data to within the limits of experimental uncertainty.

The results suggest that within the framework of the continuum model the choking criterion for a one-dimensional two-phase flow corresponds to a physical condition wherein pressure waves cannot be transmitted upstream of the choking region. This is consistent with the classic result for ideal gas flow.

The full-non-equilibrium theory developed here has been compared with sound speeds predicted by simpler conventional models of transient two-phase flow. Comparison shows that because of the large effective bubble inertia, the homogeneous flow approximation is valid for bubbly flows.

It is suggested that the use of monochromatic acoustic waves to examine two-phase flows would provide further valuable information on the boiling and momentum transfer processes.

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APPENDIX A

JUSTIFICATION OF THE ASSUMPTION THAT THE LIQUID TEMPERATURE AT THE INTERFACE IS THE LOCAL SATURATION TEMPERATURE

Bornhorst & Hatsopolous (1967) have shown using kinetic theory that the mass flux in a liquid–vapour phase change can be expressed to a good approximation as:

$$\Gamma_G = \left(\frac{2\sigma}{2-\sigma}\right)\frac{a_i}{(2\pi r T_0)^{1/2}}\frac{\rho_G h_{GL}}{T_0}\{(T_0 - T_{iL}) - (T_0 - T_{SAT})\} \qquad [A1]$$

where we have neglected pressure differences within the system. T_0 is the bulk liquid temperature outside the interface thermal boundary layer and σ is the accommodation coefficient.

An energy balance in the liquid at the interface relates Γ_G to q_{iL} (c.f. [6]),

$$\Gamma_G = -q_{iL}a_i/h_{GL}. \quad \text{[A2]}$$

Using [A1], [A2], and expressing q_{iL} in terms of T_{iL} using [B2] and [B5] of appendix B, we can obtain the relation for T_{iL} during an acoustic wave cycle:

$$(T_0 - T_{SAT}) = (T_0 - T_{iL})\left\{1 - \left(\frac{2-\sigma}{2\sigma}\right)\left(\frac{2\pi r T_0^3 \omega \epsilon_L^2}{D_L h_{GL}^4 \rho_G^2}\right)^{1/2} e^{i\pi/4}\right\} \quad \text{[A3]}$$

which shows the interface remains effectively at saturation temperature provided the second term in the curly brackets is much less than unity. This term is analogous to the non-equilibrium parameter, β, defined by Bornhorst & Hatsopolous (1967). Thus the necessary condition that $T_{iL} = T_{SAT}$ is satisfied provided:

$$\omega \ll \left(\frac{2\sigma}{2-\sigma}\right)^2 \frac{\rho_G^2 h_{GL}^4 D_L}{2\pi r \epsilon_L^2 T_0^3}. \quad \text{[A4]}$$

The value of the accommodation coefficient σ is very uncertain particularly for transient conditions. To evaluate the R.H.S. of [A4] we have adopted a minimum value of $\sigma = 0.1$ which is consistent with values given for water in the literature survey by Theofanous *et al.* (1969). For steam and water at atmospheric pressure, condition [A4] is satisfied provided $\omega < 10^6$ Hz; at $p = 70$ bar the corresponding condition is $\omega < 10^8$ Hz.

APPENDIX B

CALCULATION OF THE DIFFUSIVE HEAT FLUX IN THE LIQUID AT THE INTERFACE

The diffusive heat flux in the liquid can be calculated approximately by solving the one-dimensional Fourier equation in the liquid close to the interface. At a distance x from the interface we have

$$\frac{\partial^2 T_L}{\partial x^2} = \frac{1}{D_L}\frac{\partial T_L}{\partial t} \quad \text{[B1]}$$

subject to the boundary conditions

$$\left.\begin{aligned} T_{iL} &\to T_0 + T'_{iL} \exp i(\omega t - kz) \text{ as } x \to 0 \\ T_{iL} &\to T_0 \text{ as } x \to \infty \end{aligned}\right\}. \quad \text{[B2]}$$

The solution of [B1] satisfying boundary conditions [B2] is

$$T_L = T_0 + T'_{iL} \exp i(\omega t - kz - K^* x) \quad \text{[B4]}$$

where

$$K^* = (\omega/2D_L)^{1/2}(1 - i).$$

The diffusive heat flux into the liquid phase is simply

$$q_{iL} = -\epsilon_L\left(\frac{\partial T_L}{\partial x}\right)_{x=0} = T'_{iL}\omega^{1/2}\epsilon_L D_L^{-1/2}\exp i(\omega t - kz + \pi/4), \quad \text{[B5]}$$

showing that the heat flux leads the driving temperature difference by 45°.

Because the interface is saturated $T'_{iL} = (\mathrm{d}T/\mathrm{d}p)_{\mathrm{SAT}}p'$, where $(\mathrm{d}T/\mathrm{d}p)_{\mathrm{SAT}}$ is the gradient of the saturation line. Using this fact, and comparing [13] and [B5] we see that

$$q'_{iL}a_i = \omega_q p' \quad \text{[B6]}$$

where ω_q is a complex frequency

$$\omega_q = \omega^{1/2}\epsilon_L D_L^{-1/2}a_i(\mathrm{d}T/\mathrm{d}p)_{\mathrm{SAT}}\exp(i\pi/4).$$